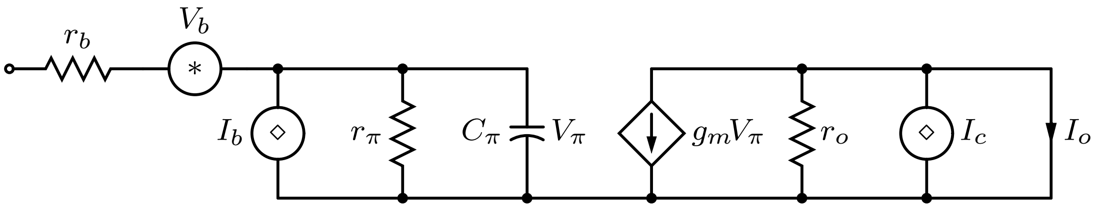
Noise Model for Bipolar Transistors

❑ Bipolar transistor equivalent circuit with noise sources: $V_b^2 = 4kTr_b$, $I_c^2 = 2qI_C$, $I_b^2 = 2qI_B + K_1 I_B/f + K_2 I_B/[1+(f/f_c)^2]$ (burst noise)



❑ Equivalent input noise voltage V_i: short-circuited input, $r_\pi \gg r_b \approx 0$, $g_m = I_C/V_T$, equivalent input noise resistance $(r_b + 1/2g_m)$

$$I_o = g_m V_b + I_c \equiv g_m V_i \quad \rightarrow \quad V_i = V_b + \frac{I_c}{g_m}$$

$$V_i^2 = V_b^2 + \frac{I_c^2}{g_m^2} = 4kT\left(r_b + \frac{2qI_C}{4kTg_m^2}\right) = 4kT\left(r_b + \frac{1}{2g_m}\right)$$

❑ Equivalent input noise current I_i: open-circuited input, V_b is excluded, equivalent input shot noise current $(I_B + K_1' I_B/f + I_C/|\beta|^2)$

$$I_o = g_m r_\pi I_b + I_c = \beta I_b + I_c \equiv \beta I_i \quad \rightarrow \quad I_i = I_b + \frac{I_c}{\beta}$$

$$I_i^2 = I_b^2 + \frac{I_c^2}{|\beta|^2} \simeq 2qI_B + \frac{K_1 I_B}{f} + \frac{2qI_C}{|\beta|^2} = 2q\left(I_B + \frac{K_1' I_B}{f} + \frac{I_C}{|\beta|^2}\right)$$

$$\simeq 2q\left(I_B + \frac{I_C}{\beta_0^2}\right) \qquad \text{(midband)} \quad \beta(s) = \frac{\beta_0}{1 + s/\omega_\beta} \simeq \beta_0$$

$$\simeq 2q\left(I_B + \frac{I_C}{\beta_0^2}\frac{f^2}{f_\beta^2}\right) \qquad (f > f_c = 50\text{ MHz}) \quad \beta(s) = \frac{\beta_0}{s/\omega_\beta}$$

$$I_B \equiv \frac{I_C}{\beta_0^2}\frac{f^2}{f_\beta^2} = \frac{I_C f^2}{f_T^2} \quad \text{for} \quad f = f_c \qquad \therefore f_c = \frac{f_T}{\sqrt{\beta_0}}$$

❑ Output noise V_{no1}^2 due only to noise currents I_{n1}, I_{nf}, I_{n-}

$$V_{no1} = I_{ni}(R_f \parallel C_f) = (I_{n1} + I_{nf} + I_{n-})\frac{R_f/sC_f}{R_f + 1/sC_f}$$

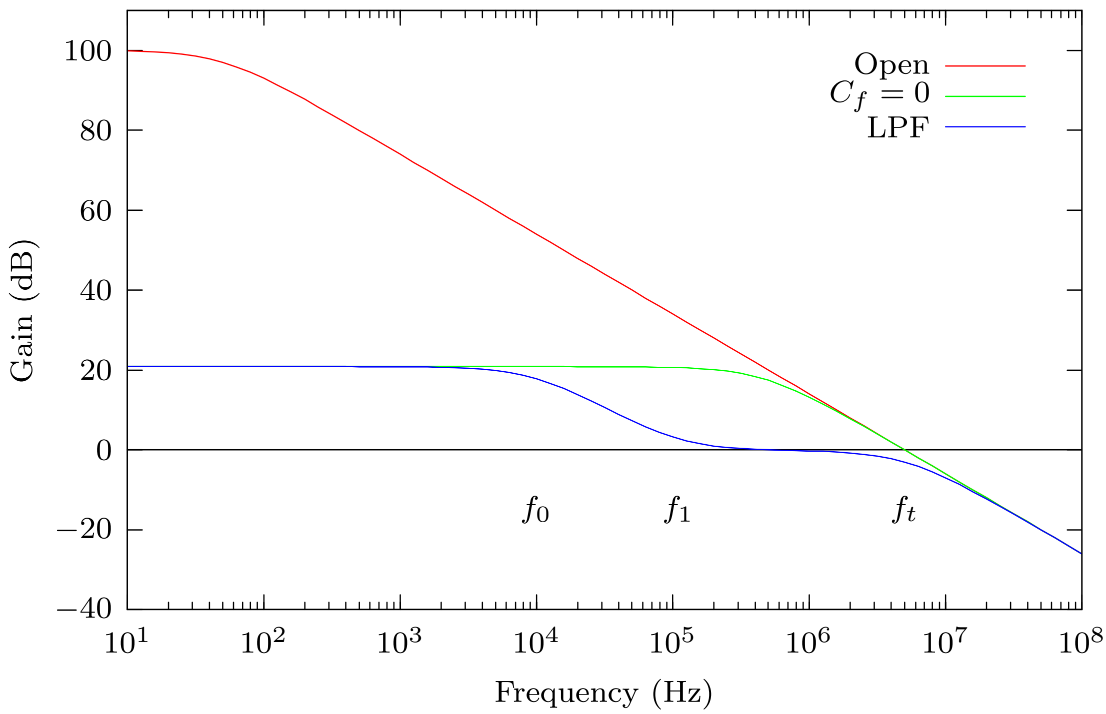
$$V_{no1}^2 = (I_{n1}^2 + I_{nf}^2 + I_{n-}^2)R_f^2 \left|\frac{1}{1 + jf/f_0}\right|^2, \quad f_0 = \frac{1}{2\pi R_f C_f}$$

❑ Output noise V_{no2}^2 due only to noise voltages V_n, V_{n2}, $I_{n+}R_2$

$$V_{no2} = V_{ni}\left(1 + \frac{R_f \parallel C_f}{R_1}\right) = V_{ni}\left(\frac{1 + R_f/R_1 + sR_fC_f}{1 + sR_fC_f}\right)$$
$$= V_{ni}(1 + R_f/R_1)\left(\frac{1 + sR_fC_f/(1 + R_f/R_1)}{1 + sR_fC_f}\right)$$
$$\equiv V_{ni}(1 + R_f/R_1)\left(\frac{1 + jf/f_1}{1 + jf/f_0}\right), \quad f_1 = (1 + R_f/R_1)f_0$$

$$V_{no2}^2 = (V_n^2 + V_{n2}^2 + I_{n+}^2R_2^2)(1 + R_f/R_1)^2\left|\frac{1 + jf/f_1}{1 + jf/f_0}\right|^2$$

❑ Gain responses of open-loop opamp, noninverting amp, active LPF



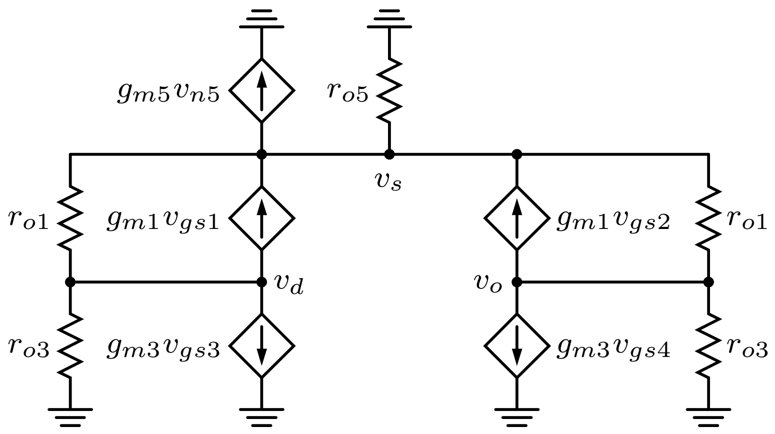
❑ Equivalent circuit of a differential amplifier for noise analysis

$(v_{gs1} + v_{n1} = v_{gs2} + v_{n2} = -v_s,\ v_d = v_{gs3} + v_{n3} = v_{gs4} + v_{n4})$

$$g_{o5}v_s + g_{o1}(v_s - v_d) + g_{o1}(v_s - v_o) - 2g_{m1}v_{gs1} = g_{m5}v_{n5}, \quad v_{gs1} = -v_s$$

$$g_{o1}(v_d - v_s) + g_{o3}v_d + g_{m1}v_{gs1} + g_{m3}v_{gs3} = 0, \quad v_{gs3} = v_d - v_{n3}$$

$$g_{o1}(v_o - v_s) + g_{o3}v_o + g_{m1}v_{gs2} + g_{m3}v_{gs4} = 0, \quad v_{gs4} = v_d - v_{n4}$$



❑ Low-frequency noises of M_5 and M_3 in the differential amplifier

$$\begin{bmatrix} 2g_{m1} + 2g_{o1} + g_{o5} & -g_{o1} & -g_{o1} \\ -(g_{m1} + g_{o1}) & g_{m3} + g_{o1} + g_{o3} & 0 \\ -(g_{m1} + g_{o1}) & g_{m3} & g_{o1} + g_{o3} \end{bmatrix} \begin{bmatrix} v_s \\ v_d \\ v_o \end{bmatrix} = \begin{bmatrix} g_{m5}v_{n5} \\ g_{m3}v_{n3} \\ g_{m3}v_{n4} \end{bmatrix}$$

$$\Delta = (g_{o1} + g_{o3})[(2g_{m1} + 2g_{o1} + g_{o5})(g_{m3} + g_{o1} + g_{o3}) - 2(g_{m1} + g_{o1})g_{o1}]$$
$$\simeq (g_{o1} + g_{o3})(2g_{m1} + 2g_{o1} + g_{o5})(g_{m3} + g_{o3})$$

$$\Delta_5 = -(g_{m1} + g_{o1})g_{m3} + (g_{m1} + g_{o1})(g_{m3} + g_{o1} + g_{o3})$$

$$\therefore \frac{v_o}{v_{n5}} = \frac{g_{m5}\Delta_5}{\Delta} = \frac{g_{m5}(g_{m1} + g_{o1})(g_{o1} + g_{o3})}{\Delta} \simeq \frac{g_{m5}}{2g_{m3}}$$

$$\Delta_3 = (2g_{m1} + 2g_{o1} + g_{o5})g_{m3} - (g_{m1} + g_{o1})g_{o1}$$

$$\therefore \frac{v_o}{v_{n3}} = \frac{-g_{m3}\Delta_3}{\Delta} \simeq \frac{-g_{m3}}{g_{o1} + g_{o3}} = -g_{m3}R_o$$